LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **APRIL 2014**

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date : 09/04/2014 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

PART – A

Answer ALL the following: (10 X 2 = 20)

1) Define a stochastic process with independent increments.
2) Define a Markov chain.
3) When do you say that a state is recurrent or transient in a Markov chain?
4) Find the period of the states 0 and 1 with transition probability matrix.
$$P = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}.$$
5) Explain n-step transition probability matrix.
6) A hospital receives on the average 3 emergency calls per hour. What is the probability that there is no call during the first 2 hours?
7) Consider the Markov chain with transition probability matrix and states 0, 1, 2:
$$\begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$$
Verify whether it is irreducible.
8) Write the PGF of the Poisson process.
9) Define a stationary distribution.
10) State the postulates of pure birth process.

PART – B

Answer any FIVE of the following: (5 X 8 = 40)

11) State and prove Chapman- Kolmogorov equation.
12) Given the transition probability matrix of a Markov chain, with states 1, 2, 3
$$P = \begin{bmatrix} 0.1 & 05 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$
$P[X_0 = i] = 1/3$, i = 1, 2, 3
Find (i) $P[X_2 = 3]$ and (ii) P [X3= 2, $X_2 = 3$, $X_1 = 3$, X_0= 2]
13) Show that the discrete queuing is a Markov chain.
14) If $i \leftrightarrow j$, then show that if i is recurrent then j is also recurrent.
15) Let $X_1(t)$ and $X_2(t)$ be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Obtain the distribution of $X_1(t) + X_2(t)$.
16) A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A in a week, the next week she buys B. However if she buys B or C, the next week she is 3 times as likely to buy A as the other cereal. Obtain the transition probability matrix.
17) Obtain the inter arrival time distribution in the Poisson process with parameter λ.

18) Consider a Markov chain with transition probability matrix

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \quad 0 < a < 1, 0 < b < 1$$

Show that $P^n = \frac{1}{a+b}\begin{bmatrix} b & a \\ b & a \end{bmatrix} + (1 - a - b)^n \begin{bmatrix} a & -a \\ -b & b \end{bmatrix}$.

PART – C

Answer any TWO of the following: (2 X 20 = 40)

19) (a) Explain the different types of classification of a stochastic process with examples.
(b) Explain how the inventory model can be viewed as a Markov chain.

20) (a) Show that i is recurrent if and only if $\sum_n P_{ii}^n = \infty$
(b) Show that in a one dimension random walk 0 is recurrent.

21) State the postulates of a Poisson process and derive the Poisson process.

22) (a) State the theorem used to find the stationary distribution. (5)

(b) Consider the Markov chain with states 0, 1, 2, 3 and the transition probability matrix

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/2 & 1/8 & 1/8 & 1/4 \end{bmatrix}$$

Check whether this chain satisfies all the conditions for getting a stationary distribution. Hence obtain the stationary probabilities.

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